2-81 An aluminum fin 1.6 mm thick surrounds a tube 2.5 cm in diameter. The length of the fin is 12.5 mm. The tube-wall temperature is 200◦C, and the environment temperature is 20◦C. The heat-transfer coefficient is 60W/m2 · ◦C. What is the heat lost by the fin?

2-84 A circumferential fin of rectangular profile is installed on a 10-cm-diameter tube maintained at 120◦C. The fin has a length of 15 cm and thickness of 2 mm. The fin is exposed to a convection environment at 23◦C with *h* = 60 W/m2 · ◦C and the fin conductivity is 120 W/m · ◦C. Calculate the heat lost by the fin expressed in watts.

2-85 A long stainless-steel rod [*k* =16 W/m · ◦C] has a square cross section 12.5 by 12.5 mm and has one end maintained at 250◦C. The heat-transfer coefficient is 40 W/m2 · ◦C, and the environment temperature is 90◦C. Calculate the heat lost by the rod.

2-86 A straight fin of rectangular profile is constructed of duralumin (94% Al, 3% Cu) with a thickness of 2.1 mm. The fin is 17 mm long, and it is subjected to a convection environment with *h*=75 W/m2 · ◦C. If the base temperature is 100◦C and the environment is at 30◦C, calculate the heat transfer per unit length of fin.

2-87 A certain internal-combustion engine is air-cooled and has a cylinder constructed of cast iron [*k* =35 Btu/h · ft · ◦F]. The fins on the cylinder have a length of 58 in and thickness of 18 in. The convection coefficient is 12 Btu/h · ft2 · ◦F. The cylinder diameter is 4 in. Calculate the heat loss per fin for a base temperature of 450◦F and environment temperature of 100◦F.

2-88 A 1.5-mm-diameter stainless-steel rod [*k* =19 W/m · ◦C] protrudes from a wall maintained at 45◦C. The rod is 12 mm long, and the convection coefficient is 500W/m2 · ◦C. The environment temperature is 20◦C. Calculate the temperature of the tip of the rod. Repeat the calculation for *h*=200 and 1500 W/m2 · ◦C.

Solution:

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi\frac{D^2}{4}}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(500)}{19(0.0015)}} = 264.9$$

$$\frac{\theta(x)}{\theta_b} = \frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh m\,(L-x)+\frac{h}{km}\sinh m\,(L-x)}{\cosh mL+\frac{h}{km}\sinh mL}$$

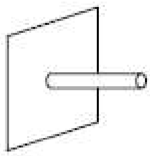
At x=L

$$\frac{T(L)-T_\infty}{T_b-T_\infty} = \frac{\cosh m\,(L-L)+\frac{h}{km}\sinh m\,(L-L)}{\cosh mL+\frac{h}{km}\sinh mL} = \frac{1}{\cosh(264.9*0.012)+\frac{500}{(19*264.9)}\sinh(264.9*0.012)} =$$

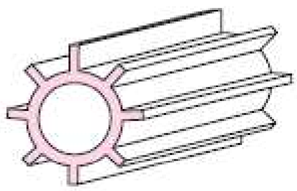
0.0756 ◦C

$T(L) = (45 - 20) * (0.0756) + 20 = 21.9$

2-89 An aluminum block is cast with an array of pin fins protruding like that shown in Figure below and subjected to room air at 20◦C. The convection coefficient between the pins and the surrounding air may be assumed to be h = 13.2 W/m2 · ◦C. The pin diameters are 2 mm and their length is 25 mm. The base of the aluminum block may be assumed constant at 70◦C. Calculate the total heat lost by an array of 15 by 15, that is, 225 fins.



2-90 A finned tube is constructed as shown in Figure below. Eight fins are installed as shown and the construction material is aluminum. The base temperature of the fins may be assumed to be 100˚C and they are subjected to a convection environment at 30˚C with h=15 W/m^2.˚C. The longitudinal length of the fins is 15 cm and the peripheral length is 2 cm. The fin thickness is 2 mm. Calculate the total heat dissipated by the finned tube. Consider only the surface area of the fins.



Solution:

k = 204 W/m.˚C, n=8, $T_b = 100$˚C, T_∞=30˚C, h=15 W/m^2.˚C, L=0.02, t=0.002

$P = 2(0.15 + 0.002) = 0.304$ m

$A_c = (0.002 * 0.15) = 0.0003$

$m = \sqrt{\frac{hP}{kA_c}} = \left(\frac{(15*0.304)}{(294*0.0003)}\right)^{\frac{1}{2}} = 8.632$

$L_c = L + \frac{t}{2} = 0.02 + \frac{0.002}{2} = 0.021$ m

$q/_{fin} = \sqrt{hPk\,A_c}\ \theta_b \tanh mL_c$

$q/_{fin} = \sqrt{15 * 0.304 * 204 * 0.0003}\,(100 - 30)\tanh(8.632 * 0.021) = 6.62$ W

$q_{total} = number\ of\ fins * {}^{q}/_{fin} = 8 * 6.62 = 53$ W.

2-92 A 2-cm-diameter glass rod 6 cm long [k =0.8 W/m · ◦C] has a base temperature of 100◦C and is exposed to an air convection environment at 20◦C. The temperature at the tip of the rod is measured as 35◦C. What is the convection heat-transfer coefficient? How much heat is lost by the rod?

2-93 A straight rectangular fin has a length of 2.5 cm and a thickness of 1.5 mm. The thermal conductivity is 55W/m · ◦C, and it is exposed to a convection environment at 20◦C and h=500 W/m2 · ◦C. Calculate the maximum possible heat loss for a base temperature of 200◦C. What is the actual heat loss?

2-94 A straight rectangular fin has a length of 3.5 cm and a thickness of 1.4 mm. The thermal conductivity is 55W/m · ◦C. The fin is exposed to a convection environment at 20◦C and h=500W/m2 · ◦C. Calculate the maximum possible heat loss for a base temperature of 150◦C. What is the actual heat loss for this base temperature?

2-95 A circumferential fin of rectangular profile is constructed of 1 percent carbon steel and attached to a circular tube maintained at 150◦C. The diameter of the tube is 5 cm, and the length is also 5 cm with a thickness of 2 mm. The surrounding air is maintained at 20◦C and the convection heat-transfer coefficient may be taken as 100 W/m2 · ◦C. Calculate the heat lost from the fin.

2-96 A circumferential fin of rectangular profile is constructed of aluminum and surrounds a 3-cm-diameter tube. The fin is 2 cm long and 1mm thick. The tube wall temperature is 200◦C, and the fin is exposed to a fluid at 20◦C with a convection heat-transfer coefficient of 80 W/m2 · ◦C. Calculate the heat loss from the fin.

2-97 A 1.0-cm-diameter steel rod [k =20 W/m · ◦C] is 20 cm long. It has one end maintained at 50◦C and the other at 100◦C. It is exposed to a convection environment at 20◦C with h=50 W/m2 · ◦C. Calculate the temperature at the center of the rod.

2-98 A circumferential fin of rectangular profile is constructed of copper and surrounds a tube having a diameter of 1.25 cm. The fin length is 6 mm and its thickness is 0.3 mm. The fin is exposed to a convection environment at 20◦C with h=55 W/m2 · ◦C and the fin base temperature is 100◦C. Calculate the heat lost by the fin.

2-99 A straight rectangular fin of steel (1% C) is 2 cm thick and 17 cm long. It is placed on the outside of a wall which is maintained at 230◦C. The surrounding air temperature is 25◦C, and the convection heat-transfer coefficient is 23W/m2 · ◦C. Calculate the heat lost from the fin per unit depth and the fin efficiency.

2-100 A straight fin having a triangular profile has a length of 5 cm and a thickness of 4 mm and is constructed of a material having k =23 W/m · ◦C. The fin is exposed to surroundings with a convection coefficient of 20W/m2 ▪ ◦C and a temperature of 40◦C. The base of the fin is maintained at 200◦C. Calculate the heat lost per unit depth of fin.

2-101 A circumferential aluminum fin is installed on a 25.4-mm-diameter tube. The length of the fin is 12.7 mm and the thickness is 1.0 mm. It is exposed to a convection environment at 30◦C with a convection coefficient of 56 W/m2 ▪ ◦C. The base temperature is 125◦C. Calculate the heat lost by the fin.

2-102 A circumferential fin of rectangular profile is constructed of stainless steel (18% Cr, 8% Ni). The thickness of the fin is 2.0 mm, the inside radius is 2.0 cm, and the length is 8.0 cm. The base temperature is maintained at 135◦C and the fin is exposed to a convection environment at 15◦C with *h*=20 W/m2 ▪ ◦C. Calculate the heat lost by the fin.

2-103 A rectangular fin has a length of 2.5 cm and thickness of 1.1 mm. The thermal conductivity is 55 W/m ▪ ◦C. The fin is exposed to a convection environment at 20◦C and *h*=500 W/m2 ▪ ◦C. Calculate the heat loss for a base temperature of 125◦C.

2-106 A stainless steel rod has a square cross section measuring 1 by 1 cm. The rod length is 8 cm, and *k* =18 W/m ▪ ◦C. The base temperature of the rod is 300◦C. The rod is exposed to a convection environment at 50◦C with *h*=45 W/m2 ▪ ◦C. Calculate the heat lost by the rod and the fin efficiency.

2-107 Copper fins with a thickness of 1.0 mm are installed on a 2.5-cm-diameter tube. The length of each fin is 12 mm. The tube temperature is 275◦C and the fins are exposed to air at 35◦C with a convection heat-transfer coefficient of 120W/m2 ▪ ◦C. Calculate the heat lost by each fin.

2-110 A circular fin of rectangular profile is attached to a 3.0-cm-diameter tube maintained at 100◦C. The outside diameter of the fin is 9.0 cm and the fin thickness is 1.0 mm. The environment has a convection coefficient of 50 W/m2 ▪ ◦C and a temperature of 30◦C. Calculate the thermal conductivity of the material for a fin efficiency of 60 percent.

Tutorial fins unuse single

3–110 Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins (k = 186 W/m · °C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ =25°C, with a heat transfer coefficient of h=40 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. *Answer:* 2639 W

3–111E Consider a stainless steel spoon (k = 8.7 Btu/h · ft · °F) partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross section of 0.08 in. 0.5 in., and extends 7 in. in the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 3 Btu/h · ft^2 · °F, determine the temperature difference across the exposed surface of the spoon handle. Neglect heat transfer from the fin tip. *Answer:* 124.6°F

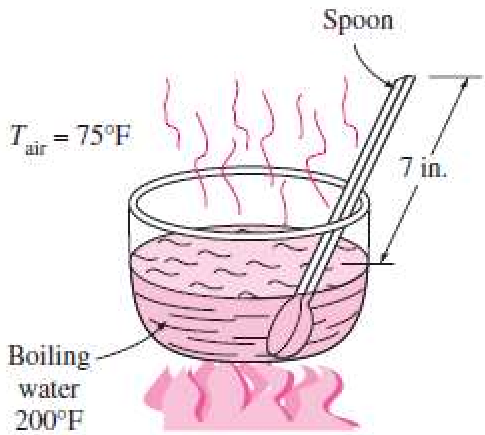
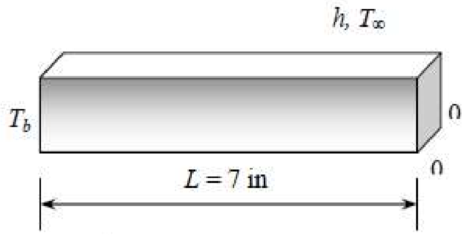


FIGURE P3–111E

Solution:

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh[m(L-x)]}{\cosh[mL]}$$

where



$$p = 2(0.5/12\text{ ft} + 0.08/12\text{ ft}) = 0.0967\text{ ft}$$

$$A_c = (0.5/12\text{ ft})(0.08/12\text{ ft}) = 0.000278\text{ ft}^2$$

$$m=\sqrt{\frac{hP}{kA_c}} \quad \sqrt{\frac{(3\text{ Btu/h.ft}^2.°\text{F})(0.0967\text{ ft})}{(8.7\text{ Btu/h.ft.°F})(0.000278\text{ ft}^2)}} = 10.95\text{ ft}^{-1}$$

N(2=0.583 ft at the tip and substituting, the tip temperature of ied to be

$$T(L) = T_\infty + (T_b - T_\infty)\frac{\cosh a(L-L)}{\cosh aL}$$

$$= 75°\text{F} + (200-75)\frac{}{\cos}\ \frac{\cosh[m(L-x)]}{\cosh[mL]}\text{F} + (200-75)\frac{1}{296} = 75.4°\text{F}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

$$\Delta T = T_b - T_{tip} = (200-75.4)°\text{F} = \mathbf{124.6°F}$$

3–116 A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins (k = 237 W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m^2 · °C. Determine the rate of heat transfer from the surface for a 1m * 1m section of the plate. Also determine the overall effectiveness of the fins.

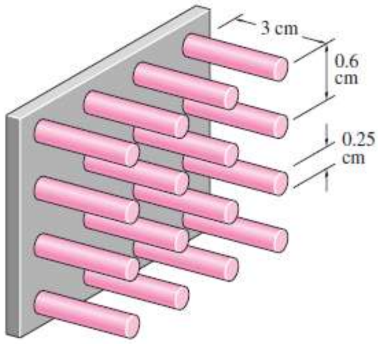


FIGURE P3–116

Solution:

$$L_c = L + \frac{D}{4} \text{ for cylindrical fin}$$

$$L_c = 0.03 + \frac{0.0025}{4} = 0.0306 \text{ m}$$

$$P = \pi D = \pi(0.0025) = 0.00785 \text{ m}^2$$

$$A_c = \frac{\pi D^2}{4} = \frac{\pi(0.0025)^2}{4} = 4.91 * 10^{-6} \text{ m}^2$$

$$mL_c = \sqrt{\frac{hP}{kA_c}}L_c = \sqrt{\frac{h\pi D}{k\frac{\pi D^2}{4}}}L_c = \sqrt{\frac{4h}{kD}}L_c = \sqrt{\frac{4(35)}{(237)(0.0025)}}0.0306 = 0.4703$$

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{\tanh 0.4703}{0.4703} = 0.93$$

$$no.of\ fin = \frac{1\ m^2}{(0.006\ m)(0.006\ m)} = 27777$$

$$A_f = 27777(\pi D L_c) = 27777[\pi(0.0025 * 0.0306)] = 6.675 \text{ m}^2$$

$$A_{un,f} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left(\frac{\pi(0.0025)^2}{4}\right) = 0.863 \text{ m}^2$$

$$q_{f,max} = A_f h(T_b - T_\infty) = 6.675 * 35(100 - 30) = 16353.75 \text{ W}$$

$$q_f = \eta_f q_{f,max} = 0.93 * 16353.75 = 15209 \text{ W}$$

$$q_{un,f} = hA_{un,f}(T_b - T_\infty) = 35 * 0.863(100 - 30) = 2114.35 \text{ W}$$

$$q_{total,f} = q_{un,f} + q_f = 2114.35 + 15209 = 17323.35 \text{ W}$$

$$A_{no,f} = 1\text{m x1 m=1m}^2$$

$$q_{no,f} = hA_{no,f}(T_b - T_\infty) = 35 * 1(100 - 30) = 2450 \text{ W}$$

$$\varepsilon_f = \frac{q_f}{q_{no,f}} = \frac{17323.35}{2450} = 7.07$$

$$\varepsilon_f = \frac{q_f}{q_{no,f}} = \frac{\eta_f A_f h(T_b - T_\infty) + hA_{un,f}(T_b - T_\infty)}{hA_{no,f}(T_b - T_\infty)}$$

$$\varepsilon_f = \frac{\eta_f A_f + A_{un,f}}{A_{no,f}}$$

Chapter 3

Transient Heat Conduction

The Lumped Capacitance Method

simple, yet common, transient conduction problem is one for which a solid experiences a sudden change in its thermal environment. Consider a hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_\infty < T_i$ (Figure 3.1). If the quenching is said to begin at time $t = 0$, the temperature of the solid will decrease for time $t > 0$, until it eventually reaches T_∞.

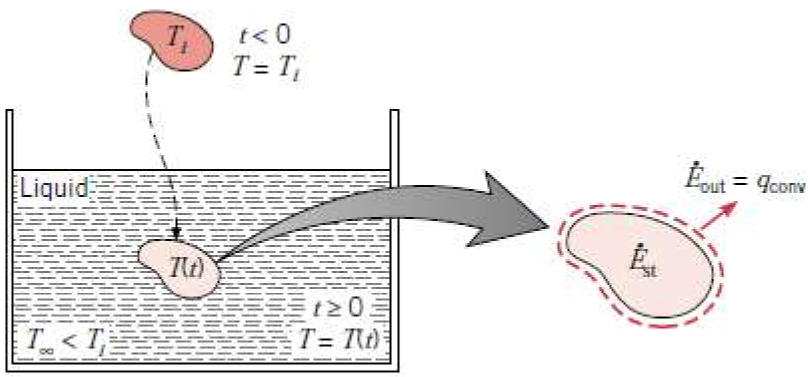


FIGURE 3.1 Cooling of a hot metal forging.

The convection heat loss from the body is evidenced as a decrease in the internal energy of the body, as shown in Figure 3-1. Thus,

$$hA_s(T - T_\infty) = -\rho Vc\frac{dT}{dt} \qquad 3.1$$

Introducing the temperature difference

$$\theta = T - T_\infty \qquad 3.2$$

And recognizing that at T_∞ constant

$\frac{dT}{dt} = \frac{d\theta}{dt}$, it followes that

$$\frac{\rho Vc}{hA_s}\frac{d\theta}{dt} = -\theta$$

Seperating variables and integrating from the initial condition, for which

B.C when $t = 0$ leads to $T(0) = T_i$

Therefore,

$$\frac{\rho Vc}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

Where

$$\theta_i = T_i - T_\infty \qquad 3.3$$

Evaluating the integrals, it follows that

$$t = \frac{\rho Vc}{hA_s} ln\frac{\theta_i}{\theta} \qquad 3.4$$

Or

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA_s}{\rho Vc}\right)t} \qquad 3.5$$

Equation 3.4 may be used to determine the time required for the solid to reach some temperature *T*, or, conversely, Equation 3.5 may be used to compute the temperature reached by the solid at some time *t*.

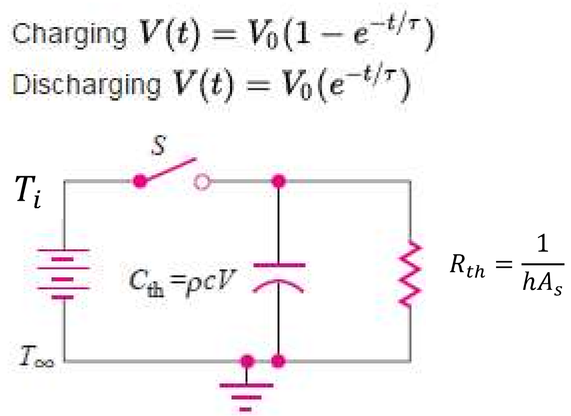


Figure 3.2: nomenclature for single-lump heat-capacity analysis.

$$\theta = \theta_i e^{-\frac{t}{\tau}}$$

$$\tau = R_{th} C_{th} = (\frac{1}{hA_s})(\rho V c) \qquad 3.6$$

Where:

τ : is the time constant.

R_{th}: is the resistance to convection heat transfer and.

C_{th}: is the *lumped thermal capacitance* of the solid.

To determine the total energy transfer q occurring up to some time t, we simply write

$$Q = \int_0^t q \ dt = hA_s \int_0^t \theta \, dt$$

$$q = hA_s \int_0^t \theta_i e^{-\frac{t}{\tau}} dt$$

$$q = hA_s(-\tau e^{-\frac{t}{\tau}} + \tau)\theta_i$$

$$q = (\rho V c)\theta_i(1 - e^{-\frac{t}{\tau}}) \qquad 3.7$$

Validity of the Lumped Capacitance Method

To develop a suitable criterion consider steady-state conduction through the plane wall of area A (Figure 3.3). Although we are assuming steady-state conditions, this criterion is readily extended to transient processes.

One surface is maintained at a temperature $T_{s,1}$ and the other surface is exposed to a fluid of temperature $T_\infty < T_{s,1}$. The temperature of this surface will be some intermediate value, $T_{s,2}$, for which $T_\infty < T_{s,2} < T_{s,1}$. Hence under steady-state conditions the surface energy balance,

$$\frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) = hA(T_{s,2} - T_\infty)$$

where k is the thermal conductivity of the solid. Rearranging, we then obtain

$$\frac{T_{s,1}-T_{s,2}}{T_{s,2}-T_\infty} = \frac{{}^{L}/_{kA}}{{}^{1}/_{hA}} = \frac{R_{cond}}{R_{conv}}$$

$$\frac{T_{s,1}-T_{s,2}}{T_{s,2}-T_\infty} = \frac{(\frac{L}{kA})}{(\frac{1}{hA})} = \frac{R_{cond}}{R_{conv}} = \frac{hV}{kA} = \frac{hL_c}{k} = B_i \qquad 3.8$$

Where

$L_c = \frac{V}{A_s}$: caracteristic length

B_i : Biot number.

Therefore, the lumped system method can used only when

$$B_i = \frac{hL_c}{k} < 0.1 \qquad 3.9$$

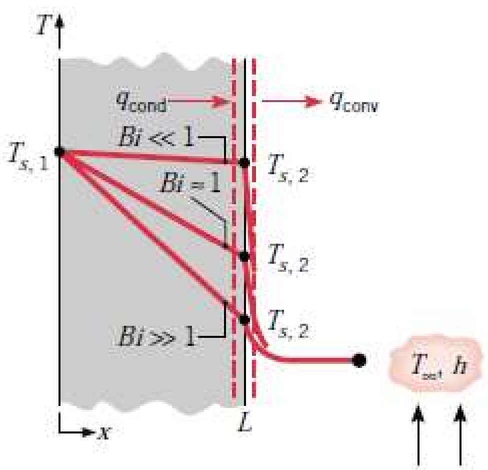


Figure 3.3 : Effect of Biot number on steady-state temperature distribution in a plane wall with surface convection.

$$\frac{hA_st}{\rho Vc} = \frac{ht}{\rho cL_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2}$$

$$\frac{hA_st}{\rho Vc} = B_i.F_o \qquad 3.10$$

Where:

$$F_o = \frac{\alpha t}{L_c^2} \text{ (Fourier number)} \qquad 3.11$$

Substituting Equation 3.10 into 3.5, we obtain

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = e^{(-B_i.F_o)} \qquad 3.12$$

Example:

A steel ball [c=0.46 kJ/kg · ◦C, k =35 W/m.◦C] 5.0 cm in diameter and initially at a uniform temperature of 450◦C is suddenly placed in a controlled environment in which the temperature is maintained at 100◦C. The convection heat-transfer coefficient is 10 W/m^2.◦C. Calculate the time required for the ball to attain a temperature of 150◦C.

Solution:

We anticipate that the lumped-capacity method will apply because of the low value of h and high value of k.We can check by using Equation (3.9):

$$B_i = \frac{hL_c}{k}$$

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{0.025}{3} = 0.00833$$

$$B_i = \frac{hL_c}{k} = \frac{10*0.00833}{35} = 0.0023 < 0.1$$

The lumped system capacitance method is valid and can used. Therefore, may can use equation 3.4:

$$t = \frac{\rho Vc}{hA_s} ln\frac{\theta_i}{\theta}$$

$t = \frac{\rho V c}{h A_s} ln \frac{T_i - T_\infty}{T - T_\infty}$

$T = 150$ ˚C, $\rho = 7800\ kg/m^3$.

T_∞ =100 ˚C, h=10 w/m^2 .˚C.

T_i =450 ˚C, c=460 J/kg. ˚C.

$t = \frac{\rho V c}{h A_s} ln \frac{T_i - T_\infty}{T - T_\infty}$

$t = \frac{\rho \frac{4}{3}\pi r^3 c}{h 4\pi r^2} ln \frac{T_i - T_\infty}{T - T_\infty}$

$t = \frac{\rho r c}{3h} ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{7800*0.025*460}{3*10} ln \frac{450-100}{150-100} = 5818.3\ s = 1.62\ h$

CHAPTER 4

Two-Dimensional, Steady-State Conduction

For steady state with no heat generation, the Laplace equation applies.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad 4.1$$

assuming constant thermal conductivity. The solution to this equation may be obtained by analytical, numerical, or graphical techniques.

The objective of any heat-transfer analysis is usually to predict heat flow or the temperature that results from a certain heat flow. The solution to Equation (4.1) will give the temperature in a two-dimensional body as a function of the two independent space coordinates x and y. Then the heat flow in the x and y directions may be calculated from the Fourier equations

$$q_x = -kA_x \frac{\partial T}{\partial x} \qquad 4.2$$